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Technical Notes

Boundary-Layer Heat Transfer from a Stretching Circular Cylinder in a Nanofluid

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Nomenclature

D_B	=	Brownian diffusion coefficient
D_T	=	thermophoretic diffusion coefficient
f	=	reduced stream function
g	=	gravitational acceleration
K	=	permeability of porous medium
k_m	=	effective thermal conductivity
Le	=	Lewis number
N_b	=	Brownian motion parameters
N_t	=	thermophoresis parameters
Nu	=	Nusselt number
p	=	pressure
q	=	wall heat flux
Re	=	Reynolds number
(r, z)	=	Cartesian coordinates
S	=	shear stress
T	=	temperature
T_w	=	wall temperature of the vertical plate
T_∞	=	ambient temperature
u, w	=	Darcy velocity components
α_m	=	thermal diffusivity of porous medium
η	=	dimensionless distance
θ	=	dimensionless temperature
μ	=	viscosity of fluid
ν	=	kinematic viscosity
ρ_f	=	fluid density
ρ_p	=	nanoparticle mass density
$(\rho c)_f$	=	heat capacity of the fluid
$(\rho c)_m$	=	effective heat capacity
$(\rho c)_p$	=	effective heat capacity of nanoparticle material
τ	=	ratio between the effective heat capacity of the nanoparticle material and that of the fluid

Subscripts

w	=	refers to condition at wall
∞	=	refers to condition far from the wall

I. Introduction

THE study of convective heat transfer in nanofluids is gaining a lot of attention. The nanofluids have many applications in the industries since materials of nanometer size have unique physical and chemical properties. Nanofluids are solid-liquid composite materials consisting of solid nanoparticles or nanofibers with sizes typically of 1–100 nm suspended in liquid. Nanofluids have attracted great interest recently because of reports of greatly enhanced thermal properties.

The characteristics of flow and heat transfer of a viscous and incompressible fluid over flexed or continuously moving flat surfaces in a moving or a quiescent fluid are well understood. These flows occur in many manufacturing processes in modern industry, such as hot rolling, hot extrusion, wire drawing and continuous casting. For example, in many metallurgical processes such as drawing of continuous filaments through quiescent fluids and annealing and tinning of copper wires, the properties of the end product depends greatly on the rate of cooling involved in these processes. Sakiadis [1] was the first one to analyze the boundary-layer flow on continuous surfaces. Crane [2] obtained an exact solution for the boundary-layer flow of Newtonian fluid caused by the stretching of an elastic sheet moving in its own plane linearly. Tsou et al. [3] extended the research to the heat transfer phenomenon of the boundary-layer flow on a continuous moving surface. Gorla and Sidawi [4] studied the characteristics of flow and heat transfer from a continuous surface with suction and blowing. Wang [5] analyzed the flow due to a stretching circular cylinder in a stagnant ambient viscous, incompressible, Newtonian fluid. Chamkha et al. [6] studied the heat transfer to a stretching cylinder moving in a stagnant free stream of an incompressible, Newtonian fluid. Ishak et al. [7] analyzed the effects of uniform suction or injection on flow and heat transfer to a stretching cylinder.

We present here a similarity analysis for the problem of a steady boundary-layer flow of a nanofluid on an isothermal stretching circular cylindrical surface. The development of the velocity, temperature and concentration distributions have been illustrated for several values of nanofluid parameters, Prandtl number, Lewis number, and Reynolds number. Applications of this work may be found in film cooling, polymer fiber coating, coating of cylindrical wires etc.

II. Analysis

Consider a stretching circular cylinder of radius a moving at a velocity $w = 2cz$ in a stagnant free stream nanofluid. The physical properties of the fluid are assumed to be constant. Under such condition, the governing equations of the steady, laminar boundary-layer flow on the moving surface are given by (see [8–10]):

$$\frac{\partial(rw)}{\partial z} + \frac{\partial(ru)}{\partial r} = 0 \tag{1}$$

$$w\frac{\partial w}{\partial z} + u\frac{\partial w}{\partial r} = \nu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) \tag{2}$$

$$w\frac{\partial u}{\partial z} + u\frac{\partial u}{\partial r} = -\frac{1}{\rho_f}\frac{\partial p}{\partial r} + \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right) \tag{3}$$

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$$w\frac{\partial T}{\partial z}+u\frac{\partial T}{\partial r}=\frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\tau\left\{D_B\left(\frac{\partial C}{\partial z}\frac{\partial T}{\partial z}+\frac{\partial C}{\partial r}\frac{\partial T}{\partial r}\right)+\left(\frac{D_T}{T_\infty}\right)\left[\left(\frac{\partial T}{\partial z}\right)^2+\left(\frac{\partial T}{\partial r}\right)^2\right]\right\} \tag{4}$$

$$w\frac{\partial C}{\partial z}+u\frac{\partial C}{\partial r}=\frac{D_B}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right)+\left(\frac{D_T}{T_\infty}\right)\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{5}$$

The boundary conditions are given by

$$r=a\colon u=0,\quad w=2cz,\quad T=T_w,\quad c=C_w$$
$$r\to\infty\colon w=0,\quad T=T_\infty,\quad c=C_\infty \tag{6}$$

Proceeding with the analysis, we define the following transformations:

$$\eta=\left(\frac{r}{a}\right)^2\quad u=-ca\left(\frac{f(\eta)}{\sqrt{\eta}}\right)\quad w=2czf'(\eta)$$
$$\theta=\frac{T-T_\infty}{T_w-T_\infty}\quad \phi=\frac{c-c_\infty}{c_w-c_\infty} \tag{7}$$

Using the transformation variables defined in Eq. (7), the governing transformed equations may be written as

$$\eta f'''+f''+Re_a(ff''-f'^2)=0 \tag{8}$$

$$\frac{\theta''}{P_r}+\frac{\theta'}{P_r\eta}+N_b\theta'\phi'+N_t(\theta')^2+\frac{Re_a}{2\eta}f\theta'=0 \tag{9}$$

$$\phi''+\phi'\left(\frac{1}{2}L_eRe_af+\frac{1}{\eta}\right)+\frac{N_t}{N_b}\frac{1}{\eta}\theta'+\frac{N_t}{N_b}\theta''=0 \tag{10}$$

The transformed boundary conditions are given by

$$\eta=1\colon f=0,\quad f'=1,\quad \theta=1,\quad \phi=1$$
$$\eta\to\infty\colon f'=0,\quad \theta=0,\quad \phi=0 \tag{11}$$

In the previous equations, the primes denote differentiation with respect to η and the six parameters are defined by

$$N_t=\frac{\varepsilon(\rho c)_pD_T(T_w-T_\infty)}{(\rho c)_fT_\infty\alpha_m}\qquad N_b=\frac{\varepsilon(\rho c)_pD_B(T_w-T_\infty)}{(\rho c)_f\alpha_m}$$
$$Re_a=\frac{ca^2}{2\nu}\qquad L_e=\frac{\nu}{D_B}\qquad P_r=\frac{\nu}{\alpha}\qquad \tau=\frac{\varepsilon(\rho c)_p}{(\rho c)_f} \tag{12}$$

Here, Pr, Le, N_b and N_t denote the Prandtl number, the Lewis number, the Brownian motion parameter and the thermophoresis parameter, respectively. It is important to note that this boundary-layer problem reduces to the classical problem of flow and heat and mass transfer due to a stretching cylinder in a viscous fluid when N_b and N_t are zero.

The quantities of practical interest, in this study, are the friction factor C_f, Nusselt number Nu and the Sherwood number Sh which are defined as the following: the wall shear stress is given by

$$S_w=\mu\frac{\partial w}{\partial r}\bigg)_{r=a}=\frac{4\mu cz}{a}\mu f''(1) \tag{13}$$

The friction factor C_f is give by

$$C_f=\frac{S_w}{\frac{\rho W^2}{2}}=4\frac{z}{a}\frac{1}{Re_z}f''(1) \tag{14}$$

where $Re_z=\frac{cz^2}{2\nu}$. The local heat transfer rate (local Nusselt number) is given by

$$Nu_x=\frac{q_wx}{k(T_w-T_\infty)}=-2\sqrt{\frac{Re_z}{Re_a}}\theta'(1) \tag{15}$$

Similarly the local Sherwood number is given by

$$Sh_x=\frac{q_mx}{D_B(C_w-c_\infty)}=-2\sqrt{\frac{Re_z}{Re_a}}\phi'(1) \tag{16}$$

where q_w and q_m are wall heat and mass flux rates, respectively.

III. Results and Discussions

The nonlinear ordinary differential Eqs. (8–10), satisfying the boundary conditions (11) were integrated numerically by using the fourth-order Runge–Kutta scheme along with the shooting method for several values of the governing parameters, namely, Prandtl number (Pr), Lewis number (Le), Brownian motion parameter (N_b) and thermophoresis parameter (N_t). The value of η_∞ was chosen to be 25 for all computations. This yielded smooth boundary-layer profiles for the velocity, temperature and concentration fields. The free stream boundary conditions were satisfied. Pantokratoras [11] discussed satisfaction of free stream boundary conditions in numerical calculations. To assess the accuracy of the present results, we obtained results for the reduced friction factor $-f''(1)$ and Nusselt number $-\theta'(1)$ by ignoring the effects of N_b and N_t. These results are shown in Tables 1 and 2. A comparison of our results with literature values indicates excellent agreement and therefore our results are highly accurate.

Table 3 display the resulting values of velocity gradient $-f''(1)$, the surface heat transfer rate $-\theta'(1)$ and the mass transfer rate $-\phi'(1)$, which are proportional to the friction factor, Nusselt number and Sherwood number, respectively. The results in Table 3 indicate that effect of increasing Re_a is to increase the heat and mass transfer rates from the surface.

Figures 1–4 show the variation of heat and mass transfer rates versus N_t with N_b, Pr and Le chosen as prescribable parameters. The heat transfer rates decrease as N_b or N_t increase. The Brownian motion of particles and thermophoresis retard heat transfer rates. The

Table 1 Comparison of results for $-f''(1)$

Re_a	Present results	Ishak et al. [7]	Wang [5]	Chamkha et al. [6]
0.5	0.88700	0.8827	0.88220	0.88700
1.0	1.17923	1.1781	1.17776	1.17953
2.0	1.59448	1.5941	1.59390	1.59444
5.0	2.41755	2.4175	2.41745	2.41798
10.0	3.34467	3.3445	——	——

Table 2 Comparison of results for $-\theta'(1)$ ($N_t=N_b=0, Re_a=3$)

Pr	Present results	Chamkha et al. [6]
0.70	1.15053	1.15053
2.00	2.10654	2.10655
7.00	4.23743	4.23743

Table 3 Effect of Re on the skin friction, rates of heat and mass transfer at $Pr=10, Le=10, N_t=0.2, N_b=0.2$

Re	$-f''(1)$	$-\theta'(1)$	$-\varphi'(1)$
0.5	0.88700	0.20729	2.59062
1	1.17923	0.30363	3.39381
2	1.59448	0.44438	4.49239
5	2.41755	0.73230	6.61972
10	3.34467	1.06430	8.98354

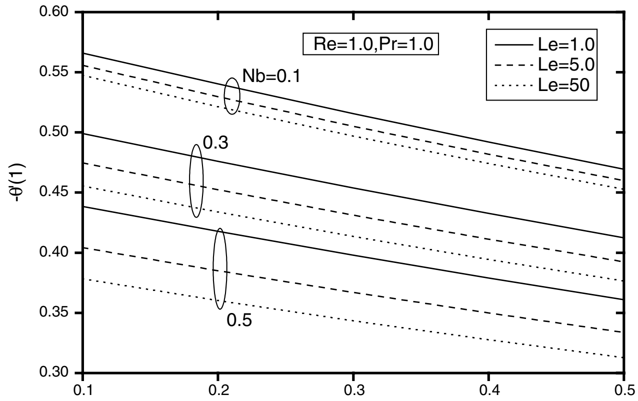

Fig. 1 Reduced local Nusselt number vs N_t.

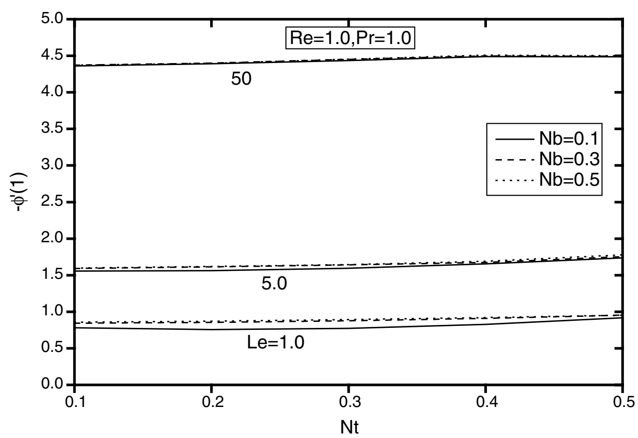

Fig. 2 Reduced local Sherwood number vs N_t.

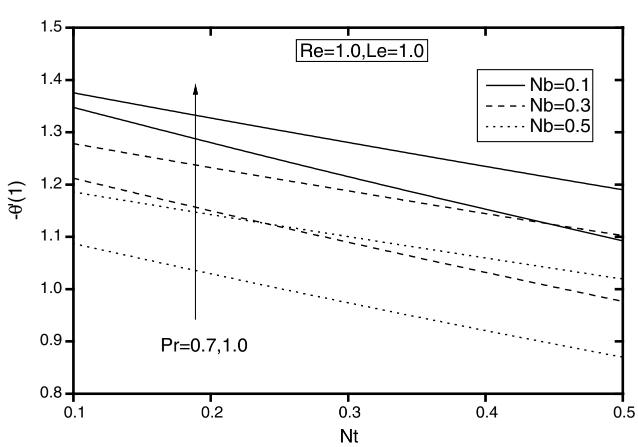

Fig. 3 Reduced local Nusselt number vs N_t.

mass transfer rates increase with N_t and decrease with N_b. The thermophoresis augments the surface mass transfer rates where as the Brownian motion of particles results in a decrease of mass transfer rates. The heat transfer rate increases as the Prandtl number Pr increases. At higher values of Pr, the thermal diffusivity decreases and therefore the heat transfer rate increases. Similarly, as Le increases, the surface mass transfer rates increase.

IV. Conclusions

This paper studied the problem of the steady boundary-layer flow of a nanofluid on a stretching isothermal circular cylinder in a stagnant free stream. The governing boundary-layer equations are solved numerically using the fourth-order Runge–Kutta scheme along with the shooting method. The development of the Nusselt number and Sherwood number as well as the temperature, concentration and velocity distributions for various values of the nanofluid parameters has been discussed and illustrated in tabular forms and graphs. The nanofluid parameters display a considerable effect on the rates of heat and mass transfer.

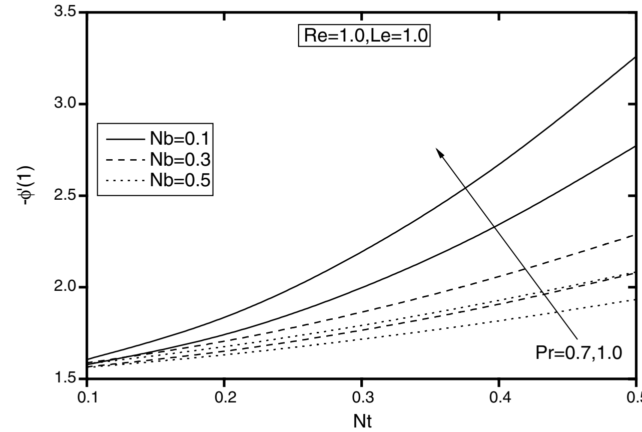

Fig. 4 Reduced local Sherwood number vs N_t.

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